The Connection Between Applied Mathematics and Deep Learning

By Manuchehr Aminian

In recent years, deep learning (DL) has inspired a myriad of advances within the scientific computing community. This subset of artificial intelligence relies on multiple components of applied mathematics, but what type of relationship do applied mathematicians have with DL? This question was the subject of a plenary talk by Yann LeCun (Facebook and New York University) at the virtual 2020 SIAM Conference on Mathematics of Data Science, which took place earlier this year. LeCun provided a brief history of machine learning (ML), highlighted the mathematical underpinnings of the field, presented both his vision and several broad open questions for ML’s future, and discussed applied math’s current relation and potential impending contributions. A 2018 SIAM News article by Gilbert Strang, entitled “The Functions of Deep Learning,” offers an introduction for those who are unfamiliar with neural networks, ML, and DL.

Stochastic Gradient Descent

LeCun immediately identified applied mathematicians’ most fundamental connection to DL: gradient descent and optimization. The search for an optimal set of parameters for a nonlinear function—with the goal of succeeding in a practical task, rather than classifying images or predicting text—comprises the heart of DL. Researchers use a special form of gradient descent to find this optimal set of parameters.

Applied mathematicians often encounter gradient descent in numerical linear algebra when they seek approximate solutions of a square linear system of equations \( Ax = b \). Finding a solution \( x \) is equivalent to finding a minimum of the quadratic function \( \frac{1}{2} x^T A x - b^T x \), over all choices of \( x \), beginning with some initial guess \( x_0 \). Researchers understand gradient descent as a process wherein they “walk down the mountain” by going in the steepest direction at every step. These actions produce a sequence of approximations \( x_n \) that is guaranteed to converge to a unique minimum for symmetric positive definite \( A \). This is admitted a very special class of functions that allows users to take the theory quite far, which is why it is taught in the classroom.

In contrast, the functions that one must minimize in DL—known as “loss functions”—are typically nonlinear and nonconvex, which makes theoretical guarantees much more challenging. Nevertheless, practitioners utilize gradient-based approaches and typically employ a modified version called stochastic gradient descent (see Figure 1). This stochastic component relates to the loss function’s evaluation; rather than using all training data to evaluate the loss, one uses a randomly selected subset of data on each iteration of gradient descent. LeCun refers to this as “walking down the mountain in a fog,” wherein each sample provides a noisy estimate of the direction. This stochastic component has found enormous practical success. “Nobody even considers anything else,” LeCun said. However, opportunities still exist for researchers to provide theory that explains this success.

The Election Interference Game

By Jenny Morber

Election interference, in which one country conspires to elect a favored candidate in another country, is a classic example of a non-cooperative game. Game theory pits two opponents against one another to achieve a goal, wherein one player’s path to success is contingent upon the strategy of the other. Therefore, when news of suspected Russian meddling in the 2016 U.S. presidential election made headlines, economist and mathematician David Dewhurst set out to model the allegations as optimal game play. Critics of game theory note that it only acknowledges this incongruity but mention an additional consideration: “If you ever wanted to look for something that is a rational actor when taken as a whole, a foreign or domestic intelligence agency is probably that,” he said. “If any group has the job of acting rationally, it is them.”

In a paper published earlier this year in Physical Review E, Dewhurst and collaborators Christopher Danforth and Peter Dodds model a two-player game in which one country, designated as “Red,” wishes to influence the outcome of a two-candidate election in another country, designated as “Blue.” Red’s goal is the election of preferred candidate \( A \), whereas Blue’s goal is an election that is free from interference—a win that is much more difficult to define. These uneven objectives put the defensive country at a disadvantage. “If you’re the FBI, you don’t want Hillary Clinton to win and you don’t want Donald Trump to win,” Dewhurst said of the 2016 election. “You just want a free and fair election. The issue is that if you take that strategy, Red always wins. So if you want to stop Red from interfering, you actually have to interfere on behalf of one of the other candidates.”

When crafting their model, the researchers considered an election between two candidates, denoted \( A \) and \( B \), that was decided by majority vote with no Electoral College. They assume that a public poll \( Z \) represents the election process at any time \( t \in \Theta \). The model’s dynamics occur in a latent space that is related to the polling process \( Z \). The function \( \phi \) is the sigmoidal function \( \phi(x) = \frac{1}{1 + e^{-x}} \). In this space, \( X < 0 \) represents values that favor candidate \( A \) and \( X > 0 \) represents values that favor candidate \( B \). The functions by which Red and Blue attempt to influence or deflect influence on the election are one-dimensional continuous-time stochastic processes, denoted by \( u_{\text{Red}}(t) \) and \( u_{\text{Blue}}(t) \). Dewhurst and his team interpret these “control policies” as expenditures on interference operations. Under the influence of both countries’ policies, the election dynamics become

\[
\begin{align*}
X_t &= F(u_{\text{Red}}(t), u_{\text{Blue}}(t)) + \text{d}W_t, \\
X_0 &= y_0.
\end{align*}
\]

Based on the assumption that \( F \) is at least twice continuously differentiable, the researchers approximate the state equation as

\[
\begin{align*}
\text{d}X_t &= \left[ u_{\text{Red}}(t) + u_{\text{Blue}}(t) \right] \text{d}t + \sigma \text{d}W_t, \\
X_0 &= y_0.
\end{align*}
\]

Red and Blue then seek to minimize the cost functions of their own control policies. \( C_{\text{Red}} \) and \( C_{\text{Blue}} \) respectively represent the running cost or benefit of conducting election interference operations, according to

\[
\begin{align*}
E_{\sigma, \phi, F} &\left[ \left( C_{\text{Red}}(u_{\text{Red}}(t), u_{\text{Blue}}(t)) + \int_0^T \left( C_{\text{Red}}(u_{\text{Red}}(t), u_{\text{Blue}}(t)) \right) \text{d}t \right) \right] \\
&\quad \text{subject to } u_{\text{Red}}(t) \in [0, 1], \quad u_{\text{Blue}}(t) \in [0, 1].
\end{align*}
\]

Here, the cost functions take the form

\[
\begin{align*}
C_{\text{Red}}(u_{\text{Red}}(t), u_{\text{Blue}}(t)) &= u_{\text{Red}}(t) \lambda_{\text{Red}}, \\
C_{\text{Blue}}(u_{\text{Red}}(t), u_{\text{Blue}}(t)) &= u_{\text{Blue}}(t) \lambda_{\text{Blue}}.
\end{align*}
\]

The notation \( \lambda \) indicates the set of all other players. Therefore, if \( i = R \), \( i = B \), and \( \lambda \) parameterizes the utility that player \( i \)
Virtual Summer Schools: Can We Make Them Work? The ongoing COVID-19 pandemic has disrupted many summer school opportunities this year. In response, a group of researchers has created a virtual “Summer School on Dynamics, Data, and the COVID-19 Pandemic” to place students in a hands-on environment. Hans and Kropf have organized the virtual school, and they hope to expand it to other topics in the future.

6 PPXPEA: Software for Exascale Computing Funded by the German Research Foundation, Softex for Exascale Computing (PPXPEA) provides a holistic approach to high-performance computing software. It ensures the efficient use of supercomputing resources, high-end supercomputers by exploring both evolutionary and adaptive research trends. Severin Reiz and Hans-Joachim Bungartz report on this multidisciplinary endeavor.

9 AN20 Panel Offers Guidance to Future Applied Mathematics Entrepreneurs The Second Joint SIAM/CAIMS Annual Meeting, which took place virtually this July, included an unique panel discussion about entrepreneurship in the applied mathematics field. Patrick Banger, Francine Gari, Jeffrey Hoffstein, and Hector Kliman described their initial career paths, detailed the challenges and rewards of founding a company, and offered advice to future entrepreneurs.

11 Professional Opportunities and Announcements

Election Interference

Continued from page 1

The election result does not deviate too far from the initial expected value. A discontinuous condition that represents this preference is given by $F_t(x) = \begin{cases} \frac{1}{2} + \frac{1}{2} \chi(x, \Delta) & \text{if } \Delta > 0 \\ \frac{1}{2} - \frac{1}{2} \chi(x, \Delta) & \text{if } \Delta < 0 \\ \frac{1}{2} & \text{if } \Delta = 0 \end{cases}$, where $\chi(x, \Delta)$ is the Heaviside step function.

Applying the dynamic programming principle to equations (2)-(4) yields the following system of coupled Hamilton-Jacobi-Bellman equations for the Red and Blue value functions:

$$V_t^R(x) = \min_{\Delta} \left[ \frac{1}{2} \chi(x, \Delta) + \frac{1}{2} \chi(x, -\Delta) - \lambda_r x^2 - \frac{1}{2} \sigma_r \lambda_r x^2 \right]$$

and

$$V_t^B(x) = \min_{\Delta} \left[ \frac{1}{2} \chi(x, \Delta) + \frac{1}{2} \chi(x, -\Delta) - \lambda_b x^2 - \frac{1}{2} \sigma_b \lambda_b x^2 \right].$$

Minimizing these equations with respect to the control variables then produces the Nash equilibrium control policies:

$$u_t^R = \begin{cases} \frac{1}{2} \chi(x, \Delta) & \text{if } \lambda_r x^2 > \frac{1}{2} \sigma_r \lambda_r x^2 \\ \frac{1}{2} \chi(x, -\Delta) & \text{if } \lambda_r x^2 < \frac{1}{2} \sigma_r \lambda_r x^2 \\ \frac{1}{2} & \text{if } \lambda_r x^2 = \frac{1}{2} \sigma_r \lambda_r x^2 \end{cases}$$

and

$$u_t^B = \begin{cases} \frac{1}{2} \chi(x, \Delta) & \text{if } \lambda_b x^2 > \frac{1}{2} \sigma_b \lambda_b x^2 \\ \frac{1}{2} \chi(x, -\Delta) & \text{if } \lambda_b x^2 < \frac{1}{2} \sigma_b \lambda_b x^2 \\ \frac{1}{2} & \text{if } \lambda_b x^2 = \frac{1}{2} \sigma_b \lambda_b x^2 \end{cases}.$$
Update from the 2020 SIAM Council Meeting

By James Crowley

As with most gatherings during this time, the annual SIAM Council meeting commenced virtually this summer. The Council met in two sessions to allow for time zone differences. In addition to the usual matters, it addressed a significant number of new business items, including reports from all of the SIAM officers and the executive director. Throughout the course of routine procedure, the Council offered suggestions for 2021 membership dues and recommended that the SIAM Board of Trustees approve all 14 activity group (SIAG) charter renewals. It deferred discussions on pricing the 2021 SIAM Annual Meeting until further information—such as the cost of a possible hybrid meeting—which would accommodate both in-person and remote attendees—is available.

During the meeting, the Council voted in favor of offering an additional two-year term to two vice presidents (VPs): VP for Education Katie Kavanagh (Clarkson University) and VP for Science Policy Anne Gelb (Dartmouth College). The group also approved Sharon Arroyo (Boeing Company) as VP for Industry, replacing retiring VP Amr El-Bakry (ExxonMobil). VPs are appointed by the SIAM president with the advice and consent of the Board and Council. The exception is the VP-at-Large, who is elected by SIAM membership.

The Council also elected Talitha Washington (Atlanta University Center Consortium Data Science Initiative) to serve on the SIAM Nominating Committee. This committee selects the slate of candidates for the fall elections of elected officers and Board and Council members. One SIAG charter renewal featured an unprecedented twist. Upon request from its officers, the SIAG on Data Mining and Analytics became the more general SIAG on Data Science (see sidebar). The group will continue to sponsor the SIAM Conference on Data Mining, and sponsor the SIAM International Conference on Data Mining, but it will now also organize the SIAM Conference on Mathematics of Data Science, which held its inaugural meeting online earlier this year.

Some major new business items are as follows:

- Creating a new position: VP for Equity, Diversity, and Inclusion (EDI)
- Voting to approve Ron Buckmire (Occidental College) as the new VP for EDI
- Establishing a Policy for Honors and Awards
- Establishing a Code of Conduct for SIAM members
- Voting to make the ad hoc Committee on Ethics a standing committee
- Establishing disclosure forms for honors, awards, and SIAM leadership—to be completed by nominees for prizes and SIAM Fellows, prize and Fellow recipients, and individuals running for SIAM offices—to certify that no known ethical issues exist
- Recommending to the Board that SIAM become a Level A partner in the annual SIAM International Conference on Mathematical Challenges in Data Science, which will take place in 2022
- Creating a Prize Canvassing Committee
- Approving a new (revised) policy on potentially offensive materials for SIAM journals and conferences, which specifically bans the Lena image
- Reviewing a significant report from the Major Awards Committee, chaired by VP-at-Large Carol Woodward (Lawrence Livermore National Laboratory), which reviewed the SIAM Prize Program.

By James Crowley

For several years, SIAM’s Fellows Canvassing Committee has encouraged the nomination of individuals from underrepresented groups for the Fellows Program. Such groups include SIAM members who work in industry, members outside of North America, and members from underrepresented demographics with respect to gender, race, and ethnicity. Canvassing committees are independent from selection committees and thus have no role in the ultimate selections. Like the Fellows Canvassing Committee, the new Prize Canvassing Committee will encourage a diverse nomination pool.

1 https://www.siam.org/membership/activity-groups/detail/data-mining-and-analytics
2 https://www.siam.org/conferences/cm/conference/mds20

VP for EDI

Creation of the VP for EDI position guarantees the presence of a voice at the Council level that promotes and ensures equity and inclusion in all facets of SIAM. The VP for EDI will be a member of the SIAM Council and will support the SIAG activity group policy and offers input on Board discussions. The position will also oversee the Minority Committee, which previously reported to the VP-at-Large and the Diversity Advisory Committee.

Policy for Honors and Awards

SIAM joined over 130 scientific and engineering societies in the Societies Consortium on Sexual Harassment in STEM2. The Societies Consortium produced a model policy, which SIAM adopted with minor modifications. When SIAM confers an honor or award upon an individual, this action reflects SIAM’s judgment that the individual’s contributions to—and effects on—the field of applied mathematics are exemplary. Unethical conduct by a current or prospective awardee can contribute to longstanding and systemic barriers in the field. Therefore, in order to more heavily emphasize what SIAM believes is best for excellence in the field rather than a single individual, SIAM will not confer an honor upon any person who has engaged in unethical conduct, or about whom a credible but undetermined question exists. The policy also allows for multiple honors, such as Fellows status, to be revoked with just cause.

Code of Conduct

Because the Policy for Honors and Awards refers to a standard of ethics, the Committee on Ethics—convened by SIAM President Lisa Fauci (Tulane University)—proposed a Code of Conduct that sets expectations for professional behavior related to harassment and issues of scientific misconduct. The Council modified and approved these guidelines, which will soon be posted online.

Disclosure Forms

It is important that SIAM promote inclusion and foster a culture of ethical behavior. While this has generally been the case in the SIAM community, the Committee on Ethics felt that a disclosure form would further hold the importance of ethical behavior in professional life. As such, candidates for leadership positions and nominees for SIAM prizes and honors must disclose any past allegations or institutional proceedings that resulted in a finding of professional misconduct, as well as current formal complaints that are related to professional conduct—even if the matter is still pending. Nominators must also certify that they are not aware of any ethical issues on the part of the nominee. These procedures will apply to prizes, SIAM Fellows, and nominations for leadership positions at SIAM.

Prize Canvassing Committee

For several years, SIAM’s Fellows Canvassing Committee has encouraged the nomination of individuals from underrepresented groups for the Fellows Program. Such groups include SIAM members who work in industry, members outside of North America, and members from underrepresented demographics with respect to gender, race, and ethnicity. Canvassing committees are independent from selection committees and thus have no role in the ultimate selections. Like the Fellows Canvassing Committee, the new Prize Canvassing Committee will encourage a diverse nomination pool.

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New SIAM Activity Group on Data Science

At the July meeting of the SIAM Council, the Council approved the creation of a new activity group on Data Science (SIAG/DATA) as a broader resource of the SIAM Activity Group on Data Mining and Analytics (SIAG/DM), which was founded in 2011. The new SIAG/DATA will aim to advance the mathematics of data science, such as it will focus on the mathematical, statistical, and computational foundations of data science, in addition to data science’s applications to other fields of science and across technology and society. SIAG/DATA will augment SIAM’s Journal on Mathematics of Data Science (SIMODS), which published its first batch of articles in 2019, by providing support for this field within SIAM. The recently initiated biannual SIAM Conference on Mathematics of Data Science (MDS) will serve as the SIAG/DATA’s flagship conference. The inaugural MDS20 took place virtually in August, with a second installment scheduled for May 2022. SIAG/DATA will continue to sponsor the annual SIAM International Conference on Data Mining (SDM). The initial slate of officers for SIAM will be Hans De Sterck (chair), Gitta Kutyniok (vice chair), and Dana Koutra (secretary); they are appointed for the duration of one year (2021). In the second half of 2021, members of the SIAG will elect new officers for the two-year period of 2022-2023. SIAM members with research interest in data science will have the opportunity to join SIAG/DATA.

By James Crowley

Revised Policy of Potentially Offensive Materials

SIAM journals have had a policy in place regarding potentially offensive materials for many years. However, as the community has grown, submittable papers or books with images of Lena, which is considered particularly offensive to many members of the SIAM community. Objectively good art has long been publicly expressed. Until now, SIAM conferences had no written policy on offensive materials. At its most recent meeting, the Council approved the adoption of a policy that explicitly bans the Lena image—along with any other offensive materials—for both journals and conferences.

Major Awards Committee

Finally, the Major Awards Committee—which is chaired by the VP-at-Large and oversees SIAM prizes—conducted a major review of the entire SIAM Prize Program, including both SIAM-wide honors and SIAG awards.

As evidenced by the numerous outcomes, this was an extremely busy and productive session for the SIAM Council. We offer our sincere thanks to Council members—especially SIAM President Lisa Fauci, who chairs the Council—for their hard work and diligence under the duress of the ongoing pandemic.
Deep Learning

Continued from page 1

Overparameterization and Networks

Turning to areas that are open to theoretical discovery, LeCun first highlighted a phenomenon that is contrary to traditional mathematical and statistical intuition—overparameterized models in DL. Mathematician John von Neumann famously said, “With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.” This sentiment reflects a common attitude among mathematicians and suggests that one should be mindful of the number of parameters in a mathematical model and the conclusions that result from fitting the model to data. Mathematicians are familiar with the perils of overparameterization from fitting high-degree polynomials. Similarly, parameter fitting and identifiability are common issues in differential equations models, especially when these models have many interacting components.

However, years of practice in DL reveal a different picture. Consider two neural networks trained for a single task. The network with more parameters—often with orders of magnitude more parameters than data points—will numerically converge in the loss function, fit the training data, and still successfully predict unseen data. In contrast, the “smaller” net frequently gets trapped in local minima in the loss surface and struggles to converge. LeCun indicated that researchers commonly understand that these overparameterized nets will automatically reduce their “rank” during training by implicit regularization, but admitted that this aspect is still a theoretical mystery. He thus suggested that the applied math community could perhaps contribute to the knowledge surrounding this problem.

The Value of an Applied Mathematics Background

DL has found particular success in the area of image processing via convolutional neural nets (CNNs). LeCun provided many examples of how CNNs can be used to talk, including medical image analysis, self-driving vehicles, and automated emergency braking systems. However, convolutional approaches in DL succeed with image-based applications because images are rectangular lattices; they face challenges when generalizing to other graph-based applications. Thus, graph convolution on the graph, one applies the convolution transform to the graph, one applies Fourier transforms to the data and filter, multiplies them, and then applies the inverse Fourier transform. Unlike computer scientists, most applied mathematicians are already quite familiar with these tools. They are consequently in a strong position to understand how applications fit in a more general theoretical landscape and are thus suited to talk to the DL community.

However, the relationship between the applied mathematics and DL communities is not one-directional. LeCun mentioned that practitioners are interested in developing DL approaches that accelerate the numerical solution of partial differential equations (PDEs). Traditionally, approaches to solving PDEs rely on the solution of finite difference or finite element discretizations of the differential equations. However, automated differentiation is related to discretization for the time and space variables, and allows for both and high-dimensional problems. In areas where careful consideration of the physics makes numerical solution difficult, one may attempt to replace the solution operator with a neural net that is trained to produce a solution but is a class of examples. While accuracy, precision, and preservation of conservation quantities are issues for such neural approximations, the potential speed-ups are quite promising.

During his talk, LeCun alluded to applications in lattice quantum chromodynamics, fluid dynamics, and astrophysics.

A Unifying Perspective

In the past, neural networks were primarily motivated by a desire to understand the living brain. Therefore, LeCun’s presentation also touched on DL approaches that mimic the ways in which humans learn, reason, and plan complex tasks. To quote LeCun, human learning is “barely supervised and rarely reinforced.” Why is this? Success with deep networks in image processing, for instance, requires thousands or even millions of labeled examples and an enormous amount of computational power for training purposes. The neural net may find success in the same class of images with which it was trained, but the process must begin again upon the introduction of a fresh class of images that were not seen during training. The user must also alert the machine to the new class of objects.

This is in stark contrast to the way that babies learn. They can recognize new objects after only seeing them a few times, and do so with very little effort and minimal external interaction. If ML’s greatest goal is to understand how humans learn, one must emulate the speed at which they do so. This direction of research is exemplified by a variety of techniques that may or may not be fit into an existing paradigm. LeCun classified these tasks under the umbrella of “self-supervised learning.”

While supervised learning and reinforcement learning have shown success in isolated tasks, LeCun believes that these paradigms will never lead to so-called “artificial general intelligence,” regardless of the scale-up of hardware capabilities. Throughout his talk, he alluded to some of the fundamental challenges that are associated with these approaches. In particular, LeCun feels that reinforcement learning will struggle to explore state space, especially when one imposes the “rarely reinforced” aspect of natural intelligence. He stated that reinforcement learning will thus make it difficult for researchers to develop a system that has “art-level intelligence,” much less human-level intelligence. As an alternative, LeCun discussed self-supervised learning via energy-based methods. He highlighted many approaches but predicted that regularized latent-variable energy-based models will be the winning framework.

LeCun concluded by addressing a pervasively misunderstood question: Is DL a science, or truly a technology? He maintained that DL is mostly the latter but argued that it is mostly the latter but argued that “just because we don’t understand it doesn’t mean we shouldn’t use it.” LeCun noted that the recent discoveries of DL and the subsequent identification of its theory can historically be quite lengthy. For example, the telescope was developed in 1608, but it took 50 years for optics to explain why it works. The steam engine appeared in 1695, but more than 100 years passed before thermodynamics could describe its function. LeCun hopes to “find the equivalent of thermodynamics for machine intelligence, or intelligence in general.”

Mathematicians will likely play a significant role in this endeavor before the DL community can reach anything that resembles a unified theory.

This article is based on Yann LeCun’s invited talk at the 2020 SIAM Conference on Mathematics of Data Science (MDS20), which occurred virtually earlier this year. LeCun’s presentation is available on SIAM’s YouTube Channel.

Manucher Aminian is an assistant professor in the Department of Mathematics and Statistics at California State Polytechnic University, Pomona. His interests include mathematical and statistical intuition, and mathematical methods in data science.

https://www.youtube.com/watch?v=9pHeQLeQIE
Virtual Summer Schools: Can We Make Them Work?

By Chris Jones and Hans Kaper

Summer schools are integral parts of students’ professional training in the mathematical sciences. Unfortunately, COVID-19 rendered in-person summer school experiences unfeasible this year. Having recently completed a six-week virtual summer school with 42 students who spanned four different time zones, we decided to reflect on the experience and summarize some of our takeaways.

The summer school on “Dynamics and Data in the COVID-19 Pandemic” was hosted by the American Institute of Mathematics (AIM), a U.S. mathematical science research institute that is supported by the National Science Foundation. The program was organized in collaboration with the Mathematics and Climate Research Network (MCRN),2 a virtual research network that engages mathematicians in climate research. Chris Jones (University of North Carolina at Chapel Hill) led the online summer school, which ran from June 22 to July 31, 2020.

The idea for the summer school was prompted in early April of 2020, and the first announcement was published on May 31. We originally planned to invite 20 advanced undergraduate and graduate students, each of whom would receive a stipend of $6,000. The school was meant to be very interactive, with structured working days consisting of large and small group discussions, one-on-one meetings, and time for individual work. All interactions were to take place online via web-based infrastructure. As the number of applications kept growing—eventually reaching well over 500—AIM and the organizing committee decided to double the allotted number of participants. This change required some organizational adjustments. The organizers hired several fully-paid mentors—junior faculty and advanced graduate students—to assist the faculty leaders, in addition to two student assistants to identify any emerging issues and facilitate interactions among students, mentors, and leaders. In total, 95 people participated: 42 students, five mentors, 10 faculty leaders, and two assistants.

The summer school also retained eight experts in mathematical epidemiology, statistics, medicine, and the information technology health industry. We all met for at least five hours each day, and everyone was on a first-name basis.

Sococo

When organizing a virtual summer school, major challenges include building a community and keeping participants engaged. An in-person environment addresses both of these issues naturally; everybody is in one place, often staying in the same building and meeting regularly, with the capacity for individual work as needed. Video conferencing is the obvious solution, but too much screen time each day—in an attempt to replicate the “being together” of an in-person school—causes exhaustion among everyone.

Keeping these challenges in mind, we sought a platform that would allow us to be present and together without a constant Zoom meeting. Sococo offers such an environment by creating a virtual workspace that effectively simulates a real one (see Figure 1). The visual layout resembles an architect’s plan without the measurements but with other amenities that make it look inviting, such as tables, chairs, and the occasional plant. The 59 participants and eight experts (each represented by an avatar) had their own private offices where they could hang out. We could leave our office doors open or closed, and anybody could come in by either clicking on the space (if open) or knocking on the door (you even heard the “tap-tap”).

The Sococo layout comprises meeting rooms, team rooms, lounges, and even a kitchen and café. There is a library to store references and a records room that holds documents of completed work, both accessible from other spaces via clickable icons. We used the café to post the next day’s schedule and a list of names with contact information, and the kitchen for sign-up sheets and the daily blog. We all met together or two or three times each day in the large “all hands” meeting room, where the daily assignments (readings, research, or videos) were posted on a bulletin board.

Sococo has a native video conferencing capability that is sufficient for small group meetings, such as an intimate conversation in a virtual office. But larger meetings require the use of Zoom, Webex, or Google Meet, which one can integrate into Sococo. When a participant begins a meeting, a Sococo pop-up dialog box invites everyone else in the room. With only one click, participants can enter the meeting with audio and video.

We used Slack, which can also be integrated into Sococo, for online chatting. Slack is particularly beneficial for a summer school like ours because it is organized by channels. Specialized interest groups that emerged throughout the program could just start a new channel.

Unfortunately, virtual platforms cannot truly replicate chance encounters in the corridor, which are unique to in-person meetings. However, we tried to engineer these encounters by stimulating visits among the participants during times such as lunch breaks. On the other hand, the virtual world offers possibilities that are not feasible—or at least not natural—in its physical counterpart. For example, several participants posted short personal videos about themselves with clickable links that other students could access from any location.

Hardware and Software

Students were required to use a tablet (iPad, Microsoft Surface, or Samsung tablet) and a digital pen or pencil. A $1,000 supplement to each stipend covered the cost of this hardware. During working sessions, students shared virtual whiteboards on Google Jamboard to jot down and explore ideas. These interactions mimicked the experience of meeting in the office and working on a blackboard together.

Daily Program

Each day, the summer school met from 11:00 a.m. to 1:30 p.m. and again from 3:00 p.m. to 5:30 p.m. EDT. Students were required to be on Sococo during these periods — in the “office” and available for meetings, either planned or spontaneous. The meeting times accommodated students in four time zones, from California to Puerto Rico. The daily program wasbracketed by “all hands” meetings at 11:00 a.m. and 5:15 p.m., and each afternoon program included a 15-minute tai-chi session to keep everybody invigorated.

Program by Week

Since the topic of the summer school was new to most participants, we split the program into two parts. Students spent the first three weeks learning and formulating questions, and the last three weeks conducting research to address these questions.

Participants identified the questions through articles (many from SIAM News’), online blogs, and simulations. We deployed various interactive exercises, such as role playing and anonymous online polling, to work through their suggestions. We also discovered that Watch2Gether—an app that allows multiple viewers to watch a video simultaneously while stopping, restarting, speed up, or slowing down the presentation—is a very useful platform for viewing online lectures. After quickly realizing that watching presentations with 50-60 people in one room was unwieldy and ineffective, we began assigning the videos as homework instead. Small groups of six or seven students then reviewed key points in Watch2Gether sessions the next morning (held concurrently with Zoom meetings).

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Figure 1. The Sococo platform creates a virtual workspace where participants can gather in team rooms, lounges, and even a kitchen and café. Here, groups meet in private offices.

1. https://aimath.org/workshops/upcoming/mcrn2020

See Summer Schools on page 7

By Chris Jones and Hans Kaper
HPC capacities is considered a feat of scientific and economic strength. And third, despite their prizes (or maybe even because of them), HPC systems are frequently easier to finance as one-time investments than the U.S. and Japan quickly reacted to this challenge with their own respective programs: SPPEXA is the DFG’s response.

SPPEXA provides a holistic approach to HPC software. It ensures the efficient use of current and upcoming high-end supercomputers by exploring both evolutionary and disruptive research threads. The following six research directions are considered most crucial: (i) computational algorithms, (ii) application software, (iii) system software and runtime libraries, (iv) programming, (v) software tools, and (vi) data management. Computational algorithms—such as fast linear solvers or eigen-solvers—are core numerical components of many large-scale application codes, including those driven by classical simulations or oriented towards data analytics. If one cannot ensure scalability for such core routines, the battle is already nearly lost. Application software acts as the “user” of HPC systems and typically appears as legacy code that researchers have developed over years or even decades. Increasing the software’s performance via a co-design that combines algorithm and performance engineering and addresses both the “systems-algorithms” and “algorithms-applications-models” interfaces is vital.

Performance engineering cannot succeed without cooperation in compilers, monitoring, code optimization, verification support, and parallelization support (like auto-tuning); this dependency collectively underlines the importance of system software, runtime libraries, and tools. Programming—including programming models—is likely the research direction where the need for a balance of evolutionary research (improving and extending existing programming models, for example) and revolutionary approaches (exploiting new programming models and language concepts, such as domain-specific languages) is most apparent. In addition, data management has always been relevant to HPC in terms of input/output or post-processing and visualization. It is therefore becoming increasingly important as more and more HPC applications are related to data.

To illustrate the wide spectrum of SPPEXA’s research impact, we highlight three exemplary subprojects:

**Algorithms:** An Exa-Scalable Two-Level Sparse Grid Approach for Higher Dimensional Problems

**Monitoring Tools:** ExtraPak

**SPPEXA is unique in numerous ways. It was the first Priority Programme that a Partnership was synchronized with two other national funding agencies with bi- and trilateral consortia:**

The Agence Nationale de la Recherche (ANR) in France and the Japan Science and Technology Agency, and the Human Brain Project, and the Enabling Cross-disciplinary Research and Engineering (ERC) on “HPC for Neuroscience.”

Two culminating events in late 2019 demonstrated the broad impact of SPPEXA’s software-related research on all associated fields: (i) an international symposium in Dresden, Germany, at which all consortia presented their results—published in Springer’s Lecture Notes in Computational Science and Engineering [1], and (ii) a trilateral workshop in Tokyo, Japan, that focused on convergence of HPC and data science. SPPEXA may have officially come to an end, but the exascale journey continues!

By Severin Reiz, Hans-Joachim Bungartz

I n 2011, the German Research Foundation (DFG) established a nation-wide Priority Programme—a coordinated funding scheme open to research consortia from all over Germany. The resulting program, called Software for Exascale Computing (SPPEXA), addresses fundamental research on various aspects of high-performance computing (HPC) software. SPPEXA involves 17 consortia, 39 research institutions, and 57 principal investigators that hail from the fields of computer science, mathematics, natural sciences, engineering sciences, and life sciences. Funding commenced in 2013, comprised two funding phases, and ended in April 2020. Overall funding from the DFG included 23 million euros. When referring to exascale computing, one typically emphasizes the corresponding high-end computer systems for several reasons:

First, the Top500 list of the 500 most powerful systems worldwide—which publishes twice a year and typically culminates in a “race to exascale”—is always celebrated as a big competition between technological approaches, vendors, and nations. Second, repeatedly providing large-scale systems, thus advancing the progress of fundamental research on various aspects of scientific computing with a focus on HPC. As such, the Top500 list is a marvel of modern-day engineering and addresses both the “systems–software” and “software–hardware” interplay.

**Figure 1.** Approximation of high-dimensional problems based on sparse grids. In this case, the grids are two-dimensional. Image adapted from [1].

**Figure 2.** Performance models generated for call paths in SWEEP3D, a neutron transport simulation. Image courtesy of [1].

**Figure 3.** Flow speeds in a mantle convection model that is computed with hierarchical hybrid grids. Image courtesy of [1].

**Figure 4.** More information about these projects and links to the code repositories are available at http://www.sppexa.de/sppexa-activities/software.html. Image provided by the authors.
The following is a short reflection from the authors of Learning LaTeX, which was first published by SIAM in 1997. The second edition appeared in 2016.

This piece is the second installment of a three-part series called "From the SIAM Bookshelf," which will periodically spotlight SIAM texts in areas of wide appeal to the greater applied mathematics and computational science communities.

In 1997, we published the first edition of Learning LaTeX to provide a useful resource for graduate students, early-career researchers, and other LaTeX beginners. We felt that it was a niche within the prevailing literature for a short and punchy introductory text that covered essential material while avoiding unnecessary detail. The internet was not all-pervading at the time, and the most readily available LaTeX information was prohibitively expensive and comprehensive, experimental textbooks. We aimed for conciseness, accessibility, and conciseness in our book that displays true LaTeX code side-by-side with the output.

Producing Learning LaTeX was a welcome opportunity to create a more restrictive format of mathematical writing. Since we were not competing with the excellent (but somewhat dry) existing LaTeX reference manuals, we could aim to be highly selective, occasionally incomplete, and lighthearted—all while attempting to avoid misleading our readers and encouraging good style (\LaTeX instead of \dots instead of \cux, and \LaTeX{} instead of \LaTeX). At merely 90 pages long, our inexpensive little text proved to be a popular resource for SIAM portfolio and continuers. We felt that there was a niche for graduate students, early-career researchers, and other LaTeX users. We felt that there was a niche for beginners. We felt that there was a niche for beginners.

Over the years, we received several suggestions by SIAM's book acquisition teams to update and restructure the book. They finally nudged us up by highlighting a review in the esteemed TUGboat journal (the Communications of the TeX Users Group), which was written 17 years after Learning LaTeX's initial publication. "I was surprised to find that the book is still in print," Boris Veytsman wrote in 2013. "Thus I ordered a copy for myself and read it." He later added that "it is a testament to the authors' efforts that this book can still be used as a first LaTeX book or as a basis for a short practical course. It provides a figure in the desired location."

On this basis, we signed the contract for a new edition and began preparing an update. Keeping a keen eye on the page counts, we cut all but the most essential new material on packages made available by the American Mathematical Society, including support for typesetting mathematical symbols and<br>

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Communication Networks and the Spread of Misinformation


S
ocial scientists Cailin O’Connor and James Owen Weatherall are faculty members at the University of California, Irvine, with secondary appointments at the university’s Institute for Mathematical Behavioral Sciences. Their new book, The Misinformation Age: How False Beliefs Spread, explains the communication networks through which nonsensical information travels—often with lasting effects—at speeds that depend on the nature of the networks in question.

The introduction and first chapter present a series of anecdotes that concern particular pieces of misinformation, while the second and third chapters analyze the properties of specific communication networks with the aid of a few simple diagrams. Many of these networks connect scientists, because their connections are comparatively easy to document. Scientific (mis/dis)belief seems to spread in much the same way as other forms of (mis/dis)belief. The fourth and final chapter of The Misinformation Age applies middle chapter methods to the role of (mis)information in public life.

The authors’ lead anecdote concerns a tale that reached 14th century Europe by way of an English knight named Sir John Mandeville. Upon his return from Asia, he spoke of a tree that bore fruit containing tiny sheep. He claimed to have tasted a third anecdote concerns the treatment of stomach ulcers, long believed to be caused by bacteria. This belief went largely unchallenged until 1954, when gastroenterologist E.D. Palmer biopsied the stomachs of more than 1,000 patients without finding evidence of bacteria. The obvious conclusion was that bacteria cannot survive in stomach acid and thus cannot cause ulcers. Instead, physicians believed that ulcers were caused by the acids themselves and could be cured by acid neutralization. In the years that followed, many ulcer patients were “successfully treated” with antibiotics, though their ulcers displayed a distressing tendency to recur. Roughly 30 years after Palmer published his results, Australian researcher J. Robin Warren detected a new strain of bacteria in biopsies near the sites of stomach ulcers. His colleague Barry Marshall isolated the new strain, proving that bacteria can in fact dwell in the human stomach. In 2005, the duo received the Nobel Prize in Medicine for convincing their fellow scientists that bacteria can and do cause stomach ulcers in humans.

A more recent fake news story appeared in September 2016 on a conservative website known as ETF News (endingthefed.com) under the headline “Pope Francis Shocked Worlds, Endorses Donald Trump for President, Releases Statement.” While the number of readers who actually believed the story is unknown, it was liked or shared 960,000 times on Facebook between the day it was posted and the election. A skeptic would argue that because the Pope is a public figure, The New York Times, Washington Post, Wall Street Journal, and any number of other media outlets would have reported any such endorsement. However, this story was but one of many false reports O’Connor and Weatherall calculate that the top 20 fake news stories in the United States were posted and the election. A skeptic would argue that because the Pope is a public figure, The New York Times, Washington Post, Wall Street Journal, and any number of other media outlets would have reported any such endorsement. However, this story was but one of many false reports O’Connor and Weatherall calculate that the top 20 fake news stories in the three months before the 2016 U.S. presidential election were liked or shared a total of 8.7 million times on Facebook, while the top 20 genuine news stories during the same period garnered only 7.3 million likes or shares.

One can visualize such models as graphs, in which each node represents an agent or group of agents and each edge represents a channel of communication between two nodes. Every node is associated with a number between 0 and 1, called a “credence,” which represents a level of certainty that action B is superior to action A. For instance, a node of credence 0.7 indicates that a particular agent is 70 percent certain that B is superior to A, while a credence that is smaller than 0.5 signifies that the agent in question favors the opposite conclusion. Weatherall and Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates. In the authors’ use mirrors Zollman’s work. Bala-Goyal models describe a collection of “agents” that are attempting to choose one of just two possible conclusions: A or B. They use information gathered by agents to analyze the networks through which scientific information propagates.
AN20 Panel Offers Guidance to Future Applied Mathematics Entrepreneurs

By Linda Sorg

Most graduates and early-career professionals in applied mathematics pursue employment in either academia or business, industry, and government. However, many are likely unfamiliar with a third possible career direction: entrepreneurship. While starting a company based on one’s strengths and interests may lead to monetary success and personal satisfaction, and can often have transformative impacts of the business world can be daunting. The Second Joint SIAM/CAIMS Annual Meeting,1 which took place virtually this July, featured an online panel discussion2 that addressed entrepreneurship within the field of applied mathematics. Speakers Patrick Bangert (Samsung SDS), Frédéric Gardi (LocalSolver), Jeffrey Hoffstein (Brown University and QuantumSafe), and Hector Klie (DeepCast.ai) discussed their individual career paths, spoke candidly about the challenges and gratifications of founding one’s own company, and offered advice to future entrepreneurs. Lalitha Venkataramanan (Schlumberger Doll Research) chaired the panel and kept conversation flowing throughout the hour-long session.

Bangers, who co-founded algorithmica technologies3 in 2005 to bridge the knowledge gap between mathematical applications and industrystyle solutions, opened the discussion. He feels that many undergraduate and graduate-level courses implicitly persuade students to become research professors, and warned that numerous academic duties—such as attracting third-party funding, performing administrative tasks, teaching, and grading student work—limit a professor’s ability to actually conduct research. While Hoffstein has enjoyed his stint as a research professor and does have adequate time to work on personal projects, he echoed Bangert’s sentiment about academic pressure. “Far too many students out there are indeed that must be a researcher,” he said. “That is completely untrue.” He encouraged students to learn as much pure and applied mathematics as possible, which increases the range of problems on which they can work. Hoffstein noted that most of today’s applied mathematics is based on what was completely pure math only 10 years ago. For example, he co-created the NTRU public key cryptosystem in 1996 and established NTRU Cryptosystems Inc. shortly thereafter; the mathematics that inspired his company were completely pure until he and his colleagues found an application.

Hoffstein is currently the chief executive officer (CEO) and co-founder of DeepCast.ai.4 He recom mended that all mathematicians who plan to start their own companies first take machine learning courses, master a pro gramming language, and become comfortable using data. Since who complete short-term internships often work with data and are thus at a distinct advantage. “A little experience is a great deal more than none,” Bangert said. “What you learn at universities is extremely important and gives you a lot of the background, but the one thing it doesn’t teach you is how to apply it in a commercial setting.” Internships with commercial businesses also teach students how to report to a boss, succeed in a hierarchical situation, and communicate with colleagues who might not have mathematical backgrounds. All of these skills are necessary when founding and leading one’s own company.

Hoffstein urged future entrepreneurs to seek out mentors with business sense. “The leadership team is considerably more important than the idea itself,” he said. “It’s a total disaster to start a company with only technical founders.” Furthermore, venture capitalists (VCs) are typically hesitant to invest in a company if none of the founders have business experience. Instead, one must also be wary of bringing in an outside CEO whose agenda might not directly align with the company’s vision.

Nevertheless, a CEO acts as a chief salesperson and no business can exist without paying customers. Bangert indicated that hired CEOs should focus on sales rather than technical matters. While some level of technical capability is useful, it is more important for an effective CEO of a fledgling company to possess a strong communicative ability and extroverted tendencies—traits that help attract VCs. Obtaining VC funding is notoriously difficult, even if a company offers a strong product.

Bangers suggested to entrepreneurs to provide advice for attendees who wish to take on an outward-facing communicative role themselves. “If you yourself are considering doing this, take a few English language and communication classes,” he said. “Be able to go out there, speak loudly and confidently, and tell a joke. If you can do some of these things in public, you’re on your way to communicat ing with a nontechnical audience.”

CAREERS IN MATHEMATICAL SCIENCES

Guiding Stress with Cable Networks and the Spider Web Problem

By Guy Bouchitté, Ornella Mattei, Graeme W. Miller, and Pierre Seppecher

When considering a junction of three or more elements that carry a current, we cannot discern in advance how much current will flow through each element; we only know from Kirchhoff’s law that the sum of the signed currents will equal zero. This is normally an advantage in an electrical circuit, as the current should flow where it is needed. However, it is disadvantageous when someone is trying to protect a circuit from current overloads or surges. Doing so requires nonlinear elements like current limiters, fuses, and surge protectors. For irrigation, this fact is disadvantageous when someone is trying to maintain a proper water-carrying capacity. Water flow surge protectors. For irrigation, this fact is disadvantageous when someone is trying to maintain a proper water-carrying capacity.

One can achieve this in a simply-connected wire network—where only one (possibly very short) wire is attached to the terminal nodes—by replacing every internal junction with a node (or at three nodes, if they are coplanar—say, in the plane). A web under tension that is as powerful argument that the result holds true even if one starts with a continuum of wires.1 In this case, the cumulative torque along each filament equals zero—i.e., the force f_i supports all the other forces f_j.

Another important question, which we call the “spider problem,” is as follows. Given a set of n balanced forces f_1, . . . , f_n at n prescribed points x_1, . . . , x_n, when does there exist a wire network under tension (which can be thought of as a spider web, and possibly have many internal junctions) that supports the forces? It suffices to examine webs when no internal nodes exist, which means that one can easily address the query with linear programming [2]. Geometrically, one internal junction at a time, we may inductively replace the wires that are connected to this internal junction with a set of wires that pairwise joint the nodes that are connected to the one they are replacing (see Figure 2, on page 105). In a sense, this is similar to the “star–delta” transformation in resistor networks, though it result only applies to the stress and not to the elastic responses of spring networks where one also monitors the displacement. A more powerful argument indicates that the result holds true even if one starts with a continuum of wires.1 In this case, the cumulative torque along each filament equals zero—i.e., the force f_i supports all the other forces f_j.

An explicit solution to this spider problem is available in two dimensions when the points x_1, . . . , x_n are coplanar—a convex polygon [2]. This two-dimensional (2D) case corresponds to the points being coplanar—say, in the horizontal plane—i.e., all f_i pointing along the plane. A web under tension then exists only if the cumulative torque Σ_j f_j x_j is in the direction of the spider web.

Figure 1. Steps in the replacement of a many-wire junction under balanced tensions, with a network localized around the junction.

No more than four tensions can meet at any node, and the network still supports the same tensions in the webs that meet it.1a, 1b. Figure 3 illustrates four wires (in black) meeting at node 0; each is presumably at the origin—and are such that they do not all lie on any two-dimensional plane localized around the node 0. Wires and superimposes a “star” network of circles whose radii are proportional to the lengths of the intervals (and 4) four rods under balancing compression that connect these four points and the origin. After superposition, a cancelation of tension occurs in one wire that goes to the origin, which one can then maintain a non-negative tension in the other wires.1c This creates three new tension junctions where five wires meet.1d. One can then replace each such junction with a web under tension, which has a maximum of four vertices at one of the new junctions. One repeats this procedure until the number of webs that meet at the origin is four or less, and then one replaces all of the webs at all of the original nodes.


2 http://www.algorithmica-technologies.com

3 https://www.deepcast.ai/
Spider Web Problem

Continued from page 9

all points \( x_i \) going clockwise around the boundary from \( x_i \) to \( x_{i+1} \) when viewed from above. One can establish this by considering the Airy stress function \( \phi(x) \) in the polygon's interior, denoted \( \Omega \). For smooth stresses \( \sigma(x) \) with \( \nabla \cdot \sigma = 0 \) in \( \Omega \), \( \phi(x) = -\int_{\partial \Omega} (\nabla \phi \cdot \nu) \ ds \). Here, \( \partial \Omega \) is the boundary of \( \Omega \), and \( \nu \) is the outward normal to \( \partial \Omega \). This is a consequence of the divergence theorem. One can modify \( \phi(x) \) by transposing \( \nabla \phi \) to \( -\nabla \phi \), and \( \sigma(x) \) is positive semidefinite in \( \Omega \) if \( \phi(x) \) is convex. For discrete wire networks under tension, this translates to \( \phi(x) \) being a convex piecewise linear (piecewise affine) function wherein the wires correspond to its edges and the wires' tension is related to the jump in \( \nabla \phi \) across the edge [4]. The convexity of \( \phi(x) \) translates to this condition on the torques. One can modify \( \phi \) inside \( \Omega \) and correspondingly change the web—as long as the tangent planes of \( \phi \) just outside the polygon's boundary remain unchanged. In particular, researchers can take \( \phi \) as the envelope of these tangent planes and find an equivalent web with no internal loops. They can then extend this result to general 2D webs and replace each convex loop with a locally open web. The area of the loops increases with each replacement until only non-convex loops remain, each of which must have internal forces directed towards every vertex where the polygon loop points inward. Thus, if \( m \) forces exist in the interior of the convex hull of the polygon's vertices, there is an open web under tension with a web that has at most \( m \) internal loops. A video that demonstrates this process is available online. 1

This type of web is less likely to be blocked by the edges of solid objects, such as the corner of a wall that may penetrate the convex hull. In the 2D open webs, the points \( x_1, x_2, \ldots, x_m \) are the vertices of a convex polygon, one can move \( x_k \) backwards and opposite to the direction \( f \) while simultaneously maintaining the polygon's convexity and therefore still have an internal force directed towards every vertex where the polygon loop points inward. Thus, if \( m \) forces exist in the interior of the convex hull of the polygon's vertices, there is an open web under tension with a web that has at most \( m \) internal loops. A video that demonstrates this process is available online. 1

One can provide a characterization of “stuck” configurations. Can researchers make progress for finite deformations? That is, given moving points \( x(t) \) and forces \( f(t) \) that are dependent on \( t \), when does there exist a single web under tension that supports these forces for some given interval of \( t \)? Here, single web means a web wherein the edges between wires change with \( t \) but the topology and wire lengths do not; all wires also remain under tension and do not collide. Another interesting problem results if some of the wires become slack or collide as \( t \) changes. Can one characterize the possible linear responses of spring networks under tension while also considering displacement at the terminal nodes? Researchers have made a complete characterization without the restriction that all springs are under tension [2]. This leads to a complete characterization of the linear elastodynamic response of mass-spring networks while allowing for internal nodes [5].

Acknowledgments: Ornella Mattei and Graeme Milton are grateful to the National Science Foundation for support through grant DMS-1814545 and the University of Toulon, where this work was initiated. Figure courtesy of [1].

References


Guy Bouchitté is an emeritus professor of mathematics at the University of Toulon. Ornella Mattei is an assistant professor in the Department of Mathematics at San Francisco State University. Graeme Milton is a distinguished professor of mathematics at the University of Utah. He is a SIAM Fellow and recipient of the SIAM Ralph E. Kleinman Prize, among other distinctions. Pierre Seppecher is a professor of mathematics at the University of Toulon.

Many open problems remain in this area of research. When can one build webs under tension that avoid given obstacles? If building such webs is possible, is there an associated algorithmic procedure? Can one provide a characterization of “stuck” configurations? Can researchers make progress for finite deformations? That is, given moving points \( x(t) \) and balanced forces \( f(t) \) that are dependent on \( t \), when does there exist a single web under tension that supports these forces for some given interval of \( t \)? Here, single web means a web wherein the edges between wires change with \( t \) but the topology and wire lengths do not; all wires also remain under tension and do not collide. Another interesting problem results if some of the wires become slack or collide as \( t \) changes. Can one characterize the possible linear responses of spring networks under tension while also considering displacement at the terminal nodes? Researchers have made a complete characterization without the restriction that all springs are under tension [2]. This leads to a complete characterization of the linear elastodynamic response of mass-spring networks while allowing for internal nodes [5].

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References


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https://icerm.brown.edu
The Cauchy-Schwarz Inequality via Springs

H ere I present a different physical implementation of the idea in [1] and [2]; the mathematical portion is exactly the same except for the notations, but I still present it for the sake of self-sufficiency. Let us connect $n$ springs end-to-end, as shown in Figure 1 for $n=3$. Initially, we forcibly hold the connections at arbitrary positions. Then we release them and let the system settle into the equilibrium configuration. In the process, potential energy decreases:

$$P_0 > P_\text{eq}. \quad (1)$$

The equality holds if and only if the system was already in equilibrium at the outset. I claim that (1) is the Cauchy-Schwarz inequality (in disguise) if the springs are Hookean, i.e., if the tension of the $i$th spring is direct proportion to its length: $F = kL_i$. Indeed, since a Hookean spring’s potential energy is $\frac{1}{2}kL_i^2$, where $k=\frac{k}{L_i}$ is the spring’s “laxness”—(1) amounts to

$$\sum \sum \sum \geq \sum \left( \sum F_i \right)^2. \quad (2)$$

By setting $\lambda = x^2$ and $\lambda F = y^2$, we get the familiar form of the Cauchy-Schwarz inequality. If the springs are non-Hookean, with $F=\frac{k}{L}L_i^2$ and $\lambda = 1$, then (1) amounts to a Holder inequality via an essentially verbatim repetition of the argument in [3].

References


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PDE2D is an exceptionally flexible and easy-to-use finite element program which solves very general partial differential equations in steady-state, time-dependent and eigenvalue partial differential equations, in 1D, 2D and 3D regions. The Windows version is now FREE with purchase of “Solving Partial Differential Equation Applications with PDE2D” (John Wiley 2018):

www.pde2d.com

Misinformations

Continued from page 8

Figure 1 (on page 8) displays three copies of a graph on six vertices. The fractions beside the nodes in $A$ are the agent’s initial credences. Light nodes correspond to agents who plan to take action $A$, whereas dark nodes correspond to agents who plan to take action $B$. The agent numbers beside the nodes in (b) indicate the number of times that the actions are successful in a series of 10 independent trials by each agent. Finally, the fractions beside the nodes in $C$ represent the updated credences obtained by application of Bayes’ rule. These credences indicate that all but one agent are fairly certain that $A$ is superior to $A$ after a single round of data gathering. The sole agent in disagreement does not update because he is not in communication with any of the agents that chose alternative $B$. A second round of testing will likely produce unanimity.

Another of the authors’ telling anecdotest concerns Lady Mary Wortley Montagu, whose husband became British ambassador to the Turkish Empire. There she encountered a practice called variolation — a primitive version of inoculation in which scratching an arm and rubbing a scar or fluid from a smallpox pustule into the wound. Though a few people did indeed catch smallpox and die, most experienced a mild form of the illness while developing immunity. A smallpox victim herself, Lady Mary suc- cessfully variolated her youngest child.

Upon returning to England, Lady Mary sought to popularize variolation among the British aristocracy but met resistance from English doctors. She turned to her friend, Lady Caroline of Ansbach, Princess of Wales, for help. Though Lady Mary’s information was accurate all along, the practice did not spread among the English nobility until Lady Caroline’s two young daughters were successfully variolated.

O’Connor and Weatherall explain Lady Caroline’s influence in terms of a sequence of graph messages that vertices (agents), arranged in a ring of six around a central “queen bee.” The latter commu- nicates with everyone else, but they each communicate with her alone. Should the queen happen to revise an opinion—as she does in Figure 2—she’s agents are likely to do so as well. Such “star networks” seldom occur by themselves but often lie hidden within larger networks.

Once identified or created, these subnet-works can be of considerable value to pro- pagandists who seek to influence important events. For instance, the Russian military appears to have made subtle use of the star network concept in their alleged attempts to influence the outcome of the 2016 US presidential election. Facebook has since revealed that Russian-produced political content reached as many as 126 million US users.

A Facebook “group” is meant to facili- tate discussion between members, while a “page” is designed for an organization or celebrity to communicate with “followers.” A “community page” lies somewhere in between. Though the creator attempts to attract followers by posting messages that are of interest to a target audience, regis- tered followers may also post messages that everyone sees.

Well before the 2016 election, the Russians apparently began to create com- munity pages of potential interest to a wide variety of existing affinity groups, including the LGBTQ community, Black Lives Matter activists, gun rights supporters, white nationalism, immigration foes, and even animal lovers. They did so by posting messages that subtly reaffirmed the target audience’s beliefs to gain trust and solidify their position as “queen bee” within a star-like communication network. Only then did they begin to inject a few tempo- rarily related fake news stories into the daily flow of community give and take.

The Misinformation Age treats extremely sensitive material in an entirely scholarly manner, with 27 pages of notes and a 36-page bibliography. It is also highly inform- ative and—at least to this reviewer—a genuinely page-turner.

References


James Case writes from Baltimore, Maryland.
Artificial Intelligence and High-Performance Computing: The Driving Forces of Tomorrow's Science
By Aparna Chandramowlishwaran

Deep learning (DL)—a specific approach to artificial intelligence (AI) that is based on neural networks—has now been recognized as one of the biggest disruptive technologies of the 21st century. As researchers proposed the first neural network with backpropagation in 1982, the popularity of neural nets rose and waned until recently. Now DL is beginning to revolutionize entire fields of science and engineering.

This transformation was ignited by developments in high-performance computing (HPC) and the emergence of graphics processing units (GPUs), which make DL feasible in practice. As we move into the next decade, HPC and AI continue to enable and drive each other at an exponential pace. HPC is pushing the limits of AI model complexities, comprising of the number of learned parameters and depth of the mathematical neural network layers used. AI is augmenting scientific accuracy while simultaneously reducing training time. Meanwhile, AI is transforming the way data is managed by its ability to seek knowledge used to begin with grand theories, and physics-based models still form the core of scientific understanding. But DL models—known as "surrogate models"—are tailored to outperform first-principles models in specific tasks, although researchers must add more data to the model to ensure its robustness. AI can also serve multiple purposes, such as producing a "trust link" with several people often leading to a domain of science.

Entrepreneurs

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Gardi, who co-founded a global optimization solver called LocalSolver in 2012, filled this role within his company. He admitted that it was difficult in the beginning because he had not studied sales or marketing in school and thus had to learn quickly. As a mathematician and an artist, they clearly understand the ability to sell and communicate, he said. "Ensure that the search and design teams are well aligned with business goals to avoid losing money by working on things that don’t matter.”

Conversation then turned to the role of creativity in promoting entrepreneurial skills. "I think universities offer most or all of the necessary components," Bangert said. "Startups are organic. Some entrepreneurs, we clearly understand the ability to sell and communicate, he said. "Ensure that the search and design teams are well aligned with business goals to avoid losing money by working on things that don’t matter.”

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When Klie himself overwhelmed with stress, he paralysed in relaxed leisure activities that pull him completely out of a work mindset. Doing so increases his efficiency when he returns to the job. Upon conclusion of the panel session, attendees had the opportunity to further converse with the experts in online break-out rooms. Despite the challenges of entrepreneurship, panelists urged participants to keep calm and stable state of mind. “A large part of my job involves being creative on a short timescale,” he said, adding that daily meditation facilitates this process. When Klie himself overwhelmed with stress, he paralysed in relaxed leisure activities that pull him completely out of a work mindset. Doing so increases his efficiency when he returns to the job. Upon conclusion of the panel session, attendees had the opportunity to further converse with the experts in online break-out rooms. Despite the challenges of entrepreneurship, panelists urged participants to keep calm and stable state of mind.

The latter's objective is to produce the same output as the physics solver (i.e., respect the convergence constraints) while accelerating the training and inference workflows across diverse and increasing AI-driven scientific workflows. Future AI frameworks also provide a higher level of abstraction and computer science at the University of California, Irvine.

References


Figure 1. A sample comparison of the traditional physics solver simulation with a physics simulation and deep learning coupled framework. The former's objective is to produce the outputs as the physics solver (i.e., respect the convergence constraints) while accelerating the training and inference workflows for specialized hardware. Knowledge bases that are tailored with the ability to dynamically adapt as the data and models are refined. This can enable efficient solutions that process data close to the source as part of application workflows, thus reducing time to solution.

Data cannot drive the next generation of science if it is inaccessible. The lack of open knowledge bases and curated repositories of data for AI applications might arguably be one of the greatest limiting factors in progress when compared to other domains. There is presently no systematic way to contrast the different proposed models for solving the same science problem. An open knowledge base of data and models may both accelerate the development of AI applications and allow for reproducibility while driving the design of new AI hardware.

Applications such as autonomous vehicles, gaming, social networks, and e-commerce are the primary motivators of current AI hardware. Knowledge bases that are tailored for science with exemplar applications could potentially bridge this gap, ultimately allowing science to drive new AI technologies and contribute to the next generation of architectures.

Here I describe only some of the core issues and opportunities that researchers may see in the future as the DL revolution. DL for science is still in its infancy, and plenty of questions will present themselves in the coming decades. Only time will reveal the extent to which DL will augment or replace existing techniques for the solution of real and complex scientific problems.